# A Directional Coupler with Very Flat Coupling

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***Abstract*—A reciprocal 4-port device is a directional coupler if any input port is matched and the device possesses either 180° rotational symmetry about two perpendicular axes, or 180° rotational symmetry about an axis combined with reflection symmetry in a plane either containing or perpendicular to the axis. In this paper an expression is derived for the equivalent admittance of such a device considered as a directional coupler. This expression is used to determine the equivalent admittance of the simplest symmetrical 2 branch coupler. Furthermore, it is shown that the coupling must be the same for all frequencies for which this junction is matched by identical 2-port matching networks connected at each port. These facts were used to build a 90° hybrid in stripline with maximum power division unbalance of 0.05 dB over the 3.7- to 4.2-GHz band.**

## I. Introduction

Recently, an expression for the equivalent admittance of a 3-fold symmetric 3-port circulator junction has been derived on the basis of symmetry considerations [1], [2]. Whereas a lossless nonreciprocal 3-port device functions as a perfect circulator if each port is matched, a lossless reciprocal 4-port device functions as a directional coupler if each port is perfectly matched. The question arises as to whether an analogous equivalent admittance can be defined for a 4-port reciprocal device with symmetry. This admittance must have the property that if a 2-port network matches into it, then the same 2-port network connected in each coupler arm will match the coupler. In this paper an expression for this admittance is derived. It is evaluated for the simplest cases of a 2-branch coupler and 2 identical coupled lines. Furthermore, the surprising result is derived that in the former case the coupling must be the same for all frequencies at which the device is matched. These facts can be used to build stripline hybrids or couplers with very flat power division or coupling for bandwidths of up to 30 percent by appropriate matching.

## II. The Equivalent Admittance of a Directional Coupler

We are considering a lossless reciprocal 4-port device with such symmetry that the device looks the same looking into any one of the four ports as the input port. An example is the two branch structure of Fig. 1. Suppose now that identical 2-port matching networks are connected at each port. This process does not alter the symmetry of the structure. The requirement that the device be a directional coupler after matching is equivalent to the requirement that two pairs of

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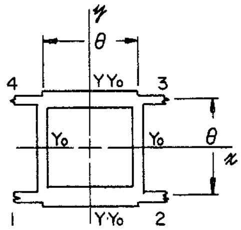


Fig. 1. A 2-branch coupler with 2 fold symmetry about the $x$ and $y$ axes. The admittance of the branch lines is $Y_0$ and of the in-line arms $Y \cdot Y_0$.

eigenreflection coefficients such as $S'_1, S'_4$ and $S'_2, S'_3$ be 180° out of phase on the unit circle [3]. Primed quantities will refer to those obtained after matching. These requirements are equivalent to the following conditions on the eigenadmittances $_jY'_1$, $_jY'_2$, $_jY'_3$, $_jY'_4$:

$$Y'_1 = -1/Y'_4, \qquad Y'_2 = -1/Y'_3. \tag{1}$$

Now if $A$, $_jB$, $_jC$, and $D$ are the elements of the transfer matrix of the matching network then

$$Y'_i = \frac{C + DY_i}{A - BY_i}, \qquad i = 1, 2, 3, 4 \tag{2}$$

where unprimed quantities refer to the eigenadmittances prior to matching. Substitution of (2) into (1) results in the following two matching conditions

$$\frac{C + DY_1}{A - BY_1} = \frac{BY_4 - A}{C + DY_4}, \qquad \frac{C + DY_2}{A - BY_2} = \frac{BY_3 - A}{C + DY_3}.$$

$$\therefore\ (A^2 + C^2) + (CD - AB)(Y_1 + Y_4) + (B^2 + D^2)Y_1Y_4 = 0$$
$$(A^2 + C^2) + (CD - AB)(Y_2 + Y_3) + (B^2 + D^2)Y_2Y_3 = 0. \tag{3}$$

Upon subtracting the bottom equation from the top one,

$$(AB - CD) = (B^2 + D^2)\left\{\frac{Y_2Y_3 - Y_1Y_4}{Y_2 + Y_3 - Y_1 - Y_4}\right\}. \tag{4}$$

This condition is identical to that on the imaginary part of a terminating admittance provided one identifies the term in brackets with this susceptance.

$$\therefore\quad Y_e = \text{Im } Y_{eq} = (Y_2Y_3 - Y_1Y_4)/(Y_2 + Y_3 - Y_1 - Y_4). \tag{5}$$

Let $R_e$ and $G_e$ be the real parts of the equivalent impedance $Z_{eq}$ and equivalent admittance $Y_{eq}$, respectively. If these exist, then the following matching conditions hold

$$1 = (A^2 + C^2)R_e \qquad 1 = (B^2 + D^2)G_e. \tag{6}$$

If these relations are substituted along with (4), (5) into the first equation of (3), then

$$\frac{1}{R_e} - \frac{1}{G_e}\{Y_e(Y_1 + Y_4) - Y_1 Y_4\} = 0 \tag{7}$$

where

$$R_e = G_e/(G_e^2 + Y_e^2). \tag{8}$$

Now (7) and (8) can be solved simultaneously to give an expression for $G_e^2$

$$G_e^2 = Y_e(Y_1 + Y_4) - Y_1 Y_4 - Y_e^2. \tag{9}$$

Equations (5) and (9) in combination yield expressions for the real and imaginary parts of the equivalent admittance. A feature which distinguishes the directional coupler case from the 3-port circulator case is that according to (9) $G_e$ is the square root of a real number. If this number is negative, an admittance will not be defined. For a 3-port circulator an equivalent admittance is always defined.

## III. The Case of Two-Coupled Lines

Suppose we first consider the simple case of two identical coupled lines of electrical length $\theta$. The eigenadmittances will be the input admittances which are obtained by imagining electric and magnetic walls to be placed on the two symmetry axes in all four combinations. The imaginary parts of these eigenadmittances become

$$Y_1 = Y_{oe}t, \qquad Y_2 = -Y_{oo}/t,$$
$$Y_3 = Y_{oo}t, \qquad Y_4 = -Y_{oe}/t$$

where $t = \tan\theta/2$ and $Y_{oe}, Y_{oo}$ are the even and odd mode admittances, respectively. If these expressions were substituted into (5) and (9) in turn, then a negative number would result for $G_e^2$ in (9) and no equivalent admittance could be defined. This is because the choice of the eigenadmittance conditions in (1) implicitly assumes that port 4 will be the isolated port whereas the port diagonally opposite the input port is the isolated port for this sort of coupler. This requires us to interchange the roles of the admittances $Y_2$ and $Y_4$ in (5) and (9). In this case,

$$Y_e = \frac{Y_3 Y_4 - Y_1 Y_2}{Y_3 + Y_4 - Y_1 - Y_2} = \frac{-Y_{oo}Y_{oe} + Y_{oo}Y_{oe}}{(Y_{oo} - Y_{oe})(t + 1/t)} = 0$$

The equivalent susceptance is identically zero. Similarly,

$$G_e^2 = Y_e(Y_1 + Y_2) - Y_1 Y_2 - Y_e^2 = Y_{oo}Y_{oe}.$$

Apparently the conductance $G_e = \sqrt{Y_{oo}Y_{oe}}$. If $Y_{oo}Y_{oe} = 1$, then the coupler will be matched independent of frequency in agreement with the usual result [4].

## IV. The 2-Branch Coupler

Next let us consider the case of the 2-branch coupler of Fig. 1. Again the eigenadmittances can be found by imagining electric and magnetic walls to be placed along the $x$ and $y$ axes in all possible combinations. These eigenadmittances are

$$_jY_1 = {_jY_0}(1 + Y)t$$
$$_jY_2 = -{_jY_0}(1 + Y)/t$$
$$_jY_3 = {_jY_0}(Yt - 1/t)$$
$$_jY_4 = {_jY_0}(t - Y/t).$$

where $Y_0$ is the characteristic admittance of the 2-branch arms while $Y \cdot Y_0$ is the characteristic admittance of the 2 in-line arms and $t = \tan(\theta/2)$. After some simple calculations it follows from (5) that

$$Y_e = \frac{Y_0(1 + Y)}{2}\left\{t - \frac{1}{t}\right\} = -Y_0(1 + Y)\cot\theta \tag{10}$$

and from (9) that

$$G_e^2 = \frac{Y_0^2(Y^2 - 1)}{4}(t + 1/t)^2$$

$$\therefore \quad G_e = \frac{Y_0\sqrt{Y^2 - 1}}{2}(t + 1/t) = \frac{Y_0\sqrt{Y^2 - 1}}{\sin\theta}. \tag{11}$$

These formulas are the ones used in a companion paper where the special case of a 2 branch 90° hybrid is treated.

The admittance given by (10) and (11) has a simple interpretation. The susceptance $Y_e$ is just that of a short circuited stub of electrical length $\theta$ and characteristic admittance $Y_0(1 + Y)$. The conductance is more complicated. At frequencies for which the line lengths of the coupler are approximately a quarter-wavelength ($\theta \simeq \pi/2$), $G_e$ will be approximated by a constant conductance $Y_0\sqrt{Y^2 - 1}$. The approximate equivalent circuit is given in Fig. 2. The equivalent admittance of a 3-port circulator is also approximated by a shunt resonator [5], [6]. Consequently, one can expect the matching networks which are useful for broadbanding circulators to be useful for broad banding this junction. A quarter-wavelength transformer shunted by a quarter-wavelength long short-circuited stub at its end will prove to be particularly useful.

## V. An Unusual Property of the 2-Branch Coupler

In this section a surprising property of the 2-branch coupler of Fig. 1 will be derived. It will be shown that the coupling of such a junction matched by 2-port matching networks at any frequency depends only on the admittance ratio $Y$ and is independent of frequency. This means that if the junction can be matched at any number of frequencies $f_1, f_2, \cdots, f_n$ by an appropriate matching network, then the coupling must be the same at all these frequencies and dependent only on $Y$. Consequently, a device with very flat coupling can be built if appropriate matching networks can be found. This result represents a generalization of a result derived in a companion paper for the case of equal power division.

Whereas the eigenreflection coefficients $S'_1, S'_4$ and $S'_2, S'_3$ must in each case be 180° out of phase for a 2-branch structure with matching networks to be a directional coupler, the coupling itself will be determined by the difference $\alpha$ in the phases of $S'_1$ and $S'_2$. It is easy to show that the ratio of the power directed to ports 2 and 3, respectively, is given by the formula

$$\left|\frac{S_{12}}{S_{13}}\right|^2 = \cot^2 \frac{\alpha}{2}. \tag{12}$$

Since $\psi'_1 = \psi'_2 + \alpha$,

$$\tan\left(-\frac{\psi'_1}{2}\right) = \tan\left(-\frac{\psi'_2}{2} - \frac{\alpha}{2}\right)$$

or,

$$Y'_1 = \frac{Y'_2 - \tan(\alpha/2)}{1 + Y'_2 \tan(\alpha/2)}$$

since $Y'_i = \tan(-\psi'_i/2)$ where $i = 1, 2, 3, 4$. In terms of the *ABCD* matrix elements of the matching network and the eigenadmittances of the unmatched junction, the last equation becomes

$$\frac{C + DY_1}{A - BY_1} = \frac{C + DY_2 - \tan(\alpha/2)\cdot(A - BY_2)}{A - BY_2 + \tan(\alpha/2)\cdot(C + DY_2)}.$$

Upon multiplying out denominators and cancelling common terms,

$$A^2 + C^2 + (B^2 + D^2)Y_1 Y_2 + (CD - AB)(Y_1 + Y_2) = \frac{Y_2 - Y_1}{\tan(\alpha/2)}.$$

Now assuming that the junction is matched by the connected matching networks,

$$G_e = \frac{1}{B^2 + D^2}, \quad R_e = \frac{1}{A^2 + C^2} = \frac{G_e}{G_e^2 + Y_e^2},$$

and

$$-\frac{Y_e}{G_e} = CD - AB.$$

Substituting these expressions into the previous equation,

$$G_e^2 + Y_e^2 + Y_1 Y_2 - Y_e(Y_1 + Y_2) = \frac{(Y_2 - Y_1)G_e}{\tan(\alpha/2)}. \tag{13}$$

If the expressions (10) and (11) for $G_e$ and $Y_e$ are substituted into (13), one obtains an equation which determines the coupling in terms of the 4 eigenadmittances of the basic junction. If the eigenadmittances of the 2-branch coupler junction are substituted, then the following coupling condition results

$$\frac{Y^2Y_0^2 - Y_0^2}{2}(t + 1/t)^2 = \frac{-Y_0}{\tan(\alpha/2)}\sqrt{\frac{Y^2Y_0^2 - Y_0^2}{4}}(t + 1/t)^2. \tag{14}$$

Notice the important result that the term containing the frequency dependence is a common factor on both sides and consequently cancels out. Equation (14) reduces to the following simple coupling condition

$$Y = \sqrt{\left|\frac{S_{12}}{S_{13}}\right|^2 + 1} \tag{15}$$

where equation (12) has been used. For a 90° hybrid $|S_{12}| = |S_{13}|$ and $Y = \sqrt{2}$. This proves the assertion made at the beginning of this section.

## VI. A Matched Directional Coupler with Very Flat Coupling

The equivalent admittance of the 2-branch coupler is approximately that of a resistance shunted by a shunt resonator as given in Fig. 2 provided that the electrical length $\theta \simeq \pi/2$. This admittance has the same form as that encountered in work with junction circulators and suggest that it should be possible to broadband this junction by connecting quarter-wavelength transformers shunted by short-circuited stubs of the same length at each port. This matching network is given in Fig. 3 where $Y_1$ is the admittance level of the transformer and $Y_2$ is the admittance level of the stub. If the *ABCD* matrix elements for this matching network are substituted into the two matching conditions containing the real and imaginary parts of the equivalent admittance given by (10) and (11), the following two equations result

$$\frac{\sin^2\theta}{Y_1^2} + (1 + Y_2/Y_1)^2\cos^2\theta = \frac{\sin\theta}{Y_0\sqrt{Y^2 - 1}} \tag{16}$$

$$\frac{\sin\theta}{Y_1} + (1 + Y_2/Y_1)\left(Y_1\sin\theta - Y_2\frac{\cos^2\theta}{\sin\theta}\right) = -(1 + \sqrt{Y}). \tag{17}$$

In these equations $Y$ is determined by the required coupling from (15), $\theta$ is determined by the required bandwidth and $Y_0$ is a free parameter which in most cases will be chosen so that $Y_0 \cdot Y \simeq 1$ in order to make a practical realization possible. Note that (16) and (17) are unchanged if $\theta$ is replaced by $180° - \theta$. Corresponding to these two electrical lengths there will be two frequencies of perfect match centered about the center frequency. If $\Delta f/f$ is the required bandwidth, then $\theta$ is given approximately by

$$\theta = \cos^{-1}\left\{\sqrt{\frac{1}{2}}\cos\left(\frac{\pi}{2}\left(1 + \frac{\Delta f}{2f}\right)\right)\right\}. \tag{18}$$

A potentially good application of this broadbanding technique is to stripline couplers and in particular to stripline 90° hybrids. Coupled line hybrids are difficult to make to close coupling tolerances because of the narrow gap between the strips. Multibranch couplers can be constructed which have a low VSWR over large bandwidths [7], [8]. However in stripline the admittance levels of certain branches assume small values requiring very narrow strips. An advantage of using 2-port matching networks is that a low VSWR can be obtained over a bandwidth of 15 percent or so with strips which are all of about the same width thereby minimizing tolerance problems. Furthermore, the coupling is exceptionally flat because of the property derived in Section V.

A stripline 3-dB coupler was constructed with $\theta$ chosen to optimize the performance over the 3.7- to 4.2-GHz band. Moreover $Y_0$ was taken to be 1 (see Fig. 1) in order to

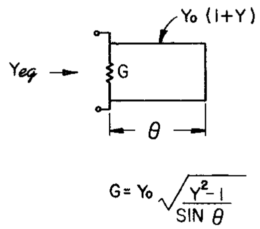


Fig. 2. The equivalent circuit for the equivalent admittance of the 2-branch coupler of Fig. 1. If the line lengths are approximately a quarter-wavelength ($\theta \simeq \pi/2$), then it is approximated by a shunt resonator with shunt conductance $G_e = Y_0\sqrt{Y^2 - 1}$.

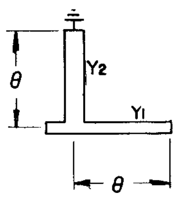


Fig. 3. A matching network consisting of a quarter-wavelength transformer and stub for broad banding the junction of Fig. 1.

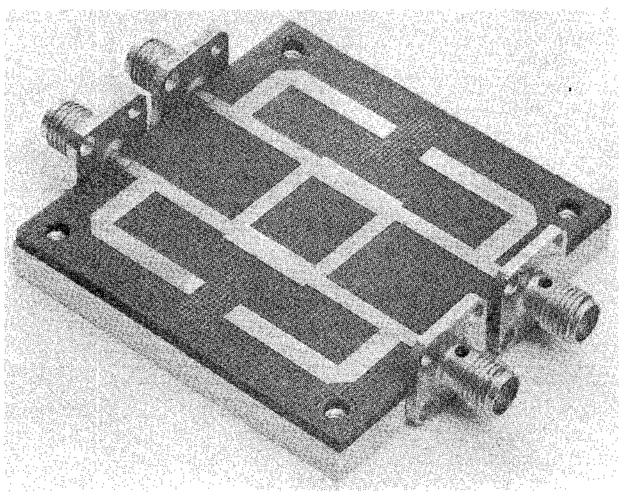

Fig. 4. A picture of a matched 90° hybrid in stripline designed to operate over the 3.7–4.2-GHz band.

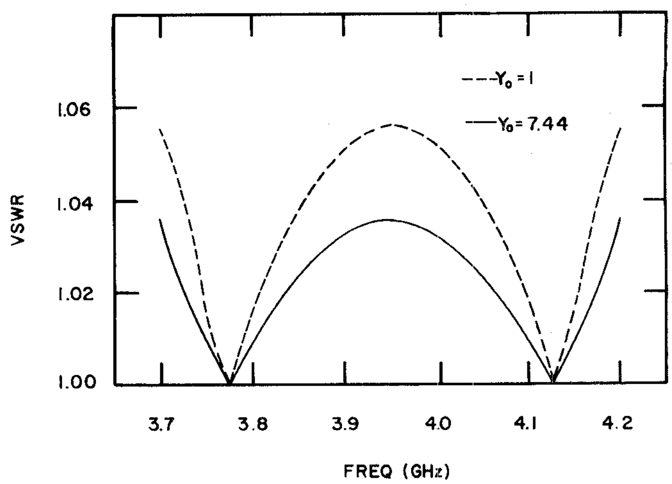


Fig. 5. Theoretical VSWR versus frequency for the hybrid of Fig. 4. The dashed curve corresponds to $Y_0 = 1$ as in Fig. 4 while the solid curve corresponds to $Y_0 = 7.44$. In the later case a quarter-wavelength transformer is sufficient and no stub is necessary.

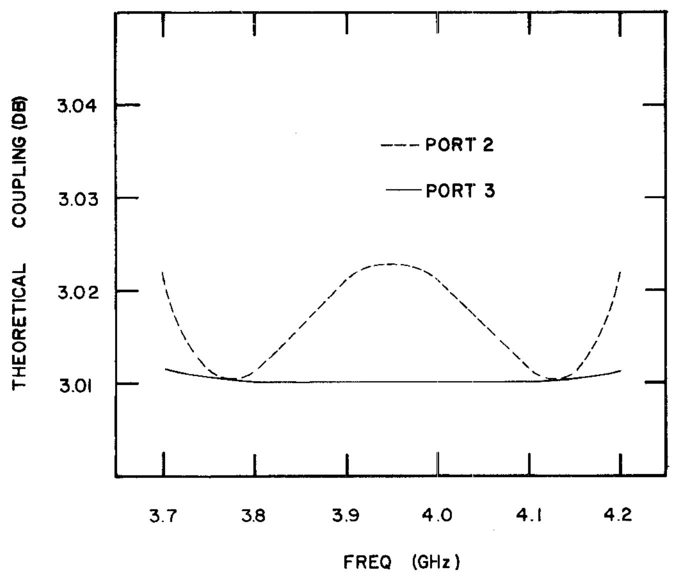


Fig. 6. Theoretical coupling to ports 2 and 3 of the hybrid pictured in Fig. 4. The power division is perfect at those frequencies for which the match is perfect and the coupling to port 2 shows a ripple unlike multibranch couplers.

produce a device with convenient strip widths. The values calculated from (16) and (17) for $Y_1$ and $Y_2$ are then 1.026 and 2.39, respectively. In the actual construction the short circuited stub of Fig. 3 was replaced by an open-circuited stub of electrical length $2\theta$ and admittance 1.195. A picture of the device is given in Fig. 4. The open-circuited stubs were folded back to make it more compact. As can be seen, all strip widths are convenient. It is important to compensate for junction effects in the initial design [9].

The theoretical VSWR is given in Fig. 5 by the dashed curve. It was calculated using expressions for the $S$ matrix elements in terms of the four eigenreflection coefficients [3]. It could also be calculated to a high degree of approximation from the equivalent admittance obtained after matching. The theoretical coupling from ports 1 to 2 and ports 2 to 3 is given in Fig. 6. Notice that the power division is perfect at those frequencies for which the device is matched in agreement with the result derived in Section V. Furthermore, the coupling to port 2 shows a ripple. This is to be contrasted with multibranch couplers where the coupling curves are parabolic. By shifting the coupling at the matched frequencies slightly away from 3.01 dB, it is in fact possible to obtain perfect power division at 4 frequencies and a maximum theoretical power unbalance between ports 2 and 3 of only 0.006 dB. The measured coupling to the in-line and branch line ports of the coupler pictured in Fig. 4 is given from 3.5 to 4.3 GHz in Fig. 7. The unbalance was within 0.05 dB from 3.7 to 4.2 GHz.

It should be emphasized that the performance can be improved by choosing a value for $Y_0$ larger than 1. In fact if $Y_0 = 7.44$, then a quarter-wavelength transformer is sufficient and a stub is no longer required to obtain a perfect match at two frequencies. The theoretical VSWR is given for

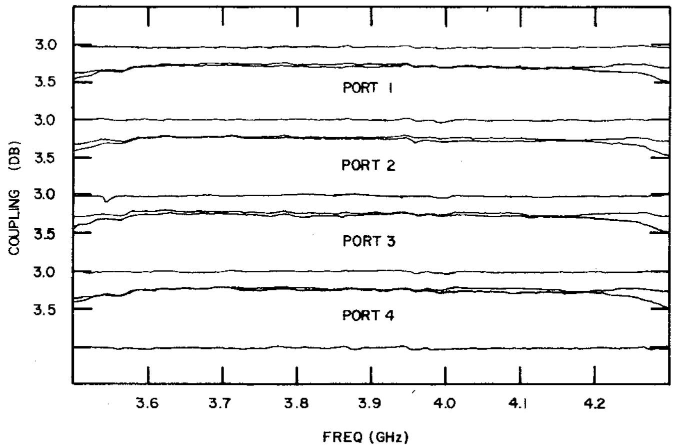


Fig. 7. The measured coupling to the four sets of in-line and branch line ports of the hybrid pictured in Fig. 4. The power division was within 0.05 dB over the 3.7–4.2-GHz band.

this case in Fig. 5. This situation corresponds to some extent to that of circulators matched with a single transformer where $G > 1$ (the conductance $G$ depends on the magnetic field strength) if two frequencies of perfect match are to be obtained. Unfortunately, it will probably be difficult to realize a matched coupler without stubs in practice because the wide strip widths required will make junction effects severe. In general the performance of couplers with lower coupling values than 3 dB will be superior to that given in Fig. 5 since the susceptance slope parameter as determined from (10) will be less.

## VII. Conclusions

An expression for the equivalent admittance of a symmetrical 4-port directional coupler has been derived. It was evaluated for the case of the 2-branch coupler where it was found to be approximated by a shunt resonator provided that $\theta \simeq \pi/2$. Furthermore the important result was derived that for this structure the coupling is independent of frequencies and will be the same at all frequencies at which it is matched. These results were used to build a stripline hybrid with very flat power division over the 3.7–4.2-GHz band by using a quarter-wavelength transformer shunted by a half-wavelength open-circuited stub as a matching element.

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